SELF-SIMILAR VARIABLE-PRESSURE COMBUSTION OF SYMMETRIC POWDER PARTICLES

Yu. A. Gostintsev

UDC 534.24

In prolonged combustion of symmetric powder particles (lamellar, cylindrical, or spherical), the temperature distribution inside a particle no longer "remembers" the thermal conditions created during the ignition process. Therefore, in principle, establishment of self-similar conditions of unsteady combustion can be expected in prolonged combustion of powder particles.

It will be seen below that such conditions are produced by the time variation of pressure if the pressure increases to a certain maximum as the particle burns up and then drops to zero.

According to the phenomenological model of unsteady powder combustion [1, 2], this problem is described by the system of equations [3, 4]

$$\frac{\partial T}{\partial t} = \frac{\varkappa}{r^s}\frac{\partial}{\partial r}\left(r^s\frac{\partial T}{\partial r}\right), \qquad 0 \leqslant r \leqslant R(t)$$
$$\frac{\partial T}{\partial r} = 0 \quad \text{for} \quad r = 0, \qquad T_s = T_s\left[p, \left(\frac{\partial T}{\partial r}\right)_{R(t)}\right] \quad \text{for} \qquad r = R(t) \tag{1}$$
$$-\frac{dR}{dt} = u\left[p, \left(\frac{\partial T}{\partial r}\right)_{R(t)}\right], \qquad p = p(t)$$

Here, the coordinate origin is located on the symmetry axis of the particle; s = 0, 1, 2 for a plate, a cylinder, or a sphere, respectively, and R(t) is the distance to the burning surface; the expressions for the surface temperature T_s and the combustion rate are assumed to be known functions of the pressure p and the surface temperature gradient in the condensed phase (k-phase).

The following dimensionless variables are introduced:

$$\tau = t\frac{u^2_0}{\varkappa}, \qquad \xi = r\frac{u_0}{\varkappa}, \qquad \vartheta = \frac{T - T_0}{T_{s0} - T_0}, \qquad w = \frac{u}{u_0}, \qquad \pi = \frac{p}{p_0}$$
$$\varphi = \left(\frac{\partial \vartheta}{\partial \xi}\right)_{\xi=\delta(\tau)}, \qquad \delta(\tau) = R(t)\frac{u_0}{\varkappa}$$

where the subscript 0 denotes the parameters characterizing the combustion of a semiinfinite powder volume at the initial temperature T_0, the surface temperature T_{s0}, and the pressure p_0.

System (1) assumes the following form:

$$\frac{\partial \vartheta}{\partial \tau} = \frac{1}{\xi^s}\frac{\partial}{\partial \xi}\left(\xi^s\frac{\partial \vartheta}{\partial \xi}\right), \qquad 0 \leqslant \xi \leqslant \delta(\tau) \tag{2}$$
$$\frac{\partial \vartheta}{\partial \xi} = 0 \quad \text{for} \quad \xi = 0, \qquad \vartheta = \vartheta_s(\pi, \varphi) \quad \text{for} \quad \xi = \delta(\tau)$$
$$-\frac{d\delta}{d\tau} = w(\pi, \varphi), \qquad \pi = \pi(\tau)$$

The initial condition concerning the temperature distribution is absent in (2), since self-similar conditions are contemplated. We shall seek the solution of the thermal conductivity equation in (2) among the class of functions

$$\vartheta = B\delta^m f(\xi/\delta) = B\delta^m f(\eta) \tag{3}$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 162-164, January-February, 1973. Original article submitted May 15, 1972.

We obtain

$$\frac{d^2f}{d\eta^2}+\frac{df}{d\eta}\left(\frac{1}{\eta}+\eta\delta\frac{d\delta}{d\tau}\right)-mf\delta\frac{d\delta}{d\tau}=0$$

It is evident that, if

$$\delta d\delta/d\tau=-4A=\text{const} \tag{4}$$

(the combustion-rate increment is inversely proportional to half the present value of the particle's thickness), the nonstationary Fourier equation from (1) is reduced to an ordinary differential equation:

$$f''+f'(s/\eta-4A\eta)+4mAf=0 \qquad (f'(0)=0,\ f(1)=1.0) \tag{5}$$

so that a self-similar temperature distribution occurs in the k-phase.

With the substitution of variables

$$f=x^{(1-s)/2}z(x),\quad x=\eta^2,\quad y=2Ax=2A\eta^2$$

(5) can be reduced to a degenerate hypergeometric equation,

$$yz''+\left(\frac{3-s}{2}-y\right)z'-\frac{1-s-m}{2}z=0$$

the solution of which is expressed in terms of Pochhammer functions ${}_1F_1(\alpha,\ \gamma,\ y)$. In terms of the initial variables f and η, this solution has the following form for a plate (s = 0) and a sphere (s = 2):

$$f_{0,2}=\eta^{1-s}\left[C_{11}F_1\left(\frac{1-s-m}{2},\frac{3-s}{2},2A\eta^2\right)+C_2(2A\eta^2)^{(s-1)/2}{}_1F_1\left(-\frac{m}{2},\frac{1+s}{2},2A\eta^2\right)\right]$$

while, for a cylinder (s = 1),

$$f_1=C_{11}F_1\left(-\frac{m}{2},1,2A\eta^2\right)+C_2\left[{}_1F_1\left(-\frac{m}{2},1,2A\eta^2\right)\ln 2A\eta^2+\right.$$
$$\left.+\sum_{k=1}^{\infty}C^k_{-m/2+k-1}\frac{(2A\eta^2)^k}{k!}\sum_{\nu=0}^{k-1}\left(\frac{1}{\nu-m/2}-\frac{2}{1+\nu}\right)\right]$$

The general solution for a self-similar temperature distribution in a lamellar, cylindrical, or spherical particle follows on the basis of finiteness of f for η = 0 and the boundary condition $f(1)$ = 1:

$$\vartheta=B\delta^m\frac{{}_1F_1(-m/2,(1+s)/2,2A\eta^2)}{{}_1F_1(-m/2,(1+s)/2,2A)} \tag{6}$$

For the surface temperature gradient $\varphi=\partial\vartheta/\partial\xi$ and the temperature ϑ_0 at the center of the particle we have from (6)

$$\varphi=-\frac{Bm}{1+s}\frac{4A}{\delta^{1-m}}\frac{{}_1F_1(1-m/2,(3+s)/2,2A)}{{}_1F_1(-m/2,(1+s)/2,2A)} \tag{7}$$

$$\vartheta_0=B\delta^m\frac{1}{{}_1F_1(-m/2,(1+s)/2,2A)} \tag{8}$$

Let us find the pressure change securing the combustion rate and the surface temperature required by (2) and (4). We assume that the dependences $u(p,\ T_0)$, $T_S(p,\ T_0)$ have been determined experimentally under steady-state conditions.

Then, the ordinary gradient conversion used for the phenomenological model in [4, 5] transforms these dependences into

$$u=u\left[p,T_s-\frac{\varkappa}{u}\left(\frac{\partial T}{\partial r}\right)_R\right],\qquad T_s=T_s\left[p,T_s-\frac{\varkappa}{u}\left(\frac{\partial T}{\partial r}\right)_R\right]$$

Let, for instance,

$$u=u_1p^{\nu}\exp\beta T_0,\quad u=D\exp(-E/RT_s)$$

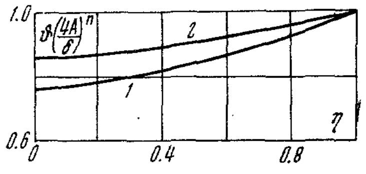

Fig. 1

Then, in terms of the dimensionless variables, we have

$$w = \pi^\nu \exp \beta (T_{s0} - T_0)(\vartheta_s - \varphi/w) \tag{9}$$

$$w = \exp \varepsilon\Delta \frac{\vartheta_s - 1}{1 + \Delta\vartheta_s} \quad \left(\varepsilon = \frac{E}{RT_{s0}},\ \Delta = \frac{T_{s0} - T_0}{T_0}\right) \tag{10}$$

Considering that $|\vartheta_s - 1| \ll 1.0$, we rewrite Eq. (10) in approximate form, as was done in [5],

$$w \simeq \vartheta^n{}_s, \quad n = \frac{\varepsilon\Delta}{1+\Delta} = \frac{E(T_{s0} - T_0)}{RT^2{}_{s0}}$$

By substituting here (3) and (4), we find the relationships between A and B and between m and n

$$m = -1/n, \quad B = (4A)^{1/n} \tag{11}$$

With an allowance for (11), expression (6) for the temperature distribution assumes the following form:

$$\vartheta = \left(\frac{4A}{\delta}\right)^{1/n} \frac{{}_1F_1(1/2n, (1+s)/2, 2A\eta^2)}{{}_1F_1(1/2n), (1+s)/2, 2A)} \tag{12}$$

As an example, Fig. 1 shows the solution of (12) for self-similar combustion of a plate (s = 0) for n = 5 and 10 and A = 0.5. We now find from (7), (9), and (11) the pressure change corresponding to these combustion conditions:

$$\pi = \left(\frac{4A}{\delta}\right)^{1/\nu} \exp\left[-\left(\frac{4A}{\delta}\right)^{1/n} \frac{B_i}{\nu}\right]$$
$$B_i = \beta(T_{s0} - T_0)\left[1 - \frac{{}_1F_1((2n+1)/2n, (3+s)/2, 2A)}{n(s+1)\,{}_1F_1(1/2n, (1+s)/2, 2A)}\right] \tag{13}$$

Analysis of (13) shows that, with a reduction in the size of the particle, the pressure increases to a certain maximum value

$$\pi_* = \left(\frac{n}{B_i}\right)^{n/\nu} \exp\left(-\frac{n}{\nu}\right) \quad \text{for} \quad \delta_* = 4A\left(\frac{B_i}{n}\right)^n$$

and then drops to zero as the particle burns up completely. For ordinary fuels, $n = E(T_{S0} - T_0)/(RT_{S0}^2) \gg 1$; as n increases, the position of the pressure maximum shifts toward smaller particle dimensions, while, for $n \to \infty$ (fuel with a constant temperature at the burning surface), we obtain the limiting law of unboundedly increasing pressure $\pi \sim \delta^{-1/\nu}$.

If we change the initial point in measuring time in the above problem and pass to infinite present dimensions of the particle, we obtain the law of pressure variation in time $\pi \sim \tau^{-1/2\nu}$, which is characteristic for self-similar combustion of a semiinfinite fuel region. This problem was considered in [1, 5].

The author is grateful to V. B. Librovich, Yu. S. Ryazantsev, S. S. Novikov, and A. G. Istratov for the discussion and their remarks.

LITERATURE CITED

1. Ya. B. Zel'dovich, "Powder combustion rate at variable pressure," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 3 (1964).
2. B. V. Novozhilov, "Unsteady combustion of powders characterized by variable surface temperature," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 1 (1967).
3. Yu. A. Gostintsev and A. D. Margolin, "Unsteady combustion of thin powder plates," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 5 (1964).
4. Yu. A. Gostintsev, "Ignition, unsteady combustion, and flame cutoff for a unitary fuel particle," Fiz. Goreniya i Vzryva, No. 3 (1971).
5. V. B. Librovich and B. V. Novozhilov, "Self-similar solutions in unsteady powder combustion rate theory," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 4 (1971).